

UNIVERSITY OF WATERLOO  
FACULTY OF ENGINEERING  
Department of Electrical &  
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ECE 204 *Numerical methods*

# Brent-Dekker method

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Brent-Dekker method

# Introduction


- In this topic, we will
  - Discuss the weaknesses of previous methods
  - Consider an algorithm by Dekker, later modified by Brent
  - We use three bracketed techniques,  
but ensure a faster convergence


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Brent-Dekker method 

## Brent-Dekker method


- Unfortunately, this is not the Bryant-Deckard method



Warner Bros. 


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Brent-Dekker method 


## The Brent-Dekker method


- The bisection method only halves the interval size
- The bracketed secant method gives a better choice approximating the root
  - The algorithm, however, generally fixes one end point, and thus is reduced to linear convergence



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Brent-Dekker method 


## The Brent-Dekker method

- Dekker's method suggests combining the bisection and bracketed secant methods
  - Simply alternating these two algorithms significantly increases the convergence of the bracketed secant method
  - Once a function near the root is either concave up or concave down, one end-point of the secant method becomes fixed
  - Inserting the bisection method unfixes this end point

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## The Brent-Dekker method

- Brent, in the 1973 text  
 “Algorithms for minimization without derivatives”  
 details an algorithm that alternates between
  - The bisection method
  - The bracketed secant method
  - Inverse parabolic interpolation
- This results in super-linear convergence
- A number of conditions are used:
  - Ideally, inverse parabolic interpolation is selected, but this technique may be determined to be sub-optimal
  - It tries to fall back on the bracketed secant method, but this, too, may be sub-optimal
  - In this case, it reverts to the bisection method

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## The Brent-Dekker method

- Brent's text is available on his website as a pdf
  - You are welcome to search for and download this text for your own personal reading
  - The end of Chapter 4 has an implementation of this algorithm in the ALGOL 60 programming language
    - You are welcome to translate this into C++ or a language of your choice
- This is the first algorithm in this course that has been written in the last half-century
  - Within the lifetime of your grandparents if not parents
  - We will see two more algorithms that fall within this time span

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Brent-Dekker method 


## The Brent-Dekker method

- The Brent-Dekker method is the basis in MATLAB for `fzero`  
<https://www.mathworks.com/help/matlab/ref/fzero.html>
  - You can view the source code by searching for the file `fzero.m`

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## Summary


- Following this topic, you now
  - Are aware of the Brent-Dekker method
  - Understand that algorithms like this exist and are available
  - Are aware that the text is available for reference



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Brent-Dekker method 

## References


[1] Richard P. Brent, "Algorithms for minimization without derivatives", Prentice-Hall, 1973




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# Acknowledgments

None so far.

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# Colophon



These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see <https://www.rbg.ca/> for more information.

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